The Casimir force and the conservation of EM energy

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Abstract *– In the gap between plates, Casimir excluded electromagnetic (EM) waves having half-wavelengths longer than the gap. But this assumption violates the conservation of EM energy. If conserved, EM energy is required to be gained at the resonant frequency of the gap to compensate for the loss from the excluded long wavelength EM radiation. By neglecting the resonant EM gap radiation, Casimir found an attractive force between the plates. However, if Casimir would have included the resonant EM radiation, he would have shown the Casimir force does not exist.*

1 Introduction

Over 50 years ago, Casimir [1] formulated the interaction between neutral conductive bodies separated from each other by an evacuated gap in terms of the zero point energy (ZPE) of quantum mechanics. Today, the ZPE more commonly called the energy of the vacuum is thought to pervade all of space, but remains controversial.

Casimir relied on Planck's derivation [2] of the law for blackbody (BB) radiation that included the ZPE. In terms of the average Planck energy E_{avg} of a harmonic oscillator,

$$E_{avg} = \frac{h\upsilon}{\left[\exp\left(h\upsilon/kT\right)-1\right]} + \frac{1}{2}h\upsilon \qquad (1)$$

where, ZPE = ½ hυ. Here, h is Planck's constant, υ is the oscillator frequency, k is Boltzmann's constant, and T is absolute temperature.

The ZPE is usually treated as a mathematical artifact of the blackbody (BB) radiation derivation and disposed of as non-physical because of the divergence at high frequency. To wit, $E_{avg} \rightarrow \infty$, as $\upsilon \rightarrow \infty$. In contrast, Einstein's derivation [3] of the radiation law excludes the ZPE,

$$E_{avg} = \frac{h\upsilon}{\left[\exp\left(h\upsilon/kT\right)-1\right]} \qquad (2)$$

The Casimir force was presumably verified [4] in tests of flat mirrors by Sparnaay. But the force measured is likely to be electrostatic and not caused by the ZPE predicted by Casimir. In fact, the force measurements [4] that ostensibly verified the ZPE were swamped [5] by the electrostatic force, as the mirrors had to be kept neutral by first touching them together before each measurement was made.

Indeed, Sparnaay's experiments were not neutral and characterized by electrostatic discharge (ESD). Because of this difficulty, the conclusion of [4] was not that the Casimir theory was confirmed, but rather that it was not contradicted.

More recently, the ZPE was derived [6] by Boyer based on classical arguments to agree with Planck. To show it is worthwhile to take the ZPE seriously, the experimental [4] verification of the Casimir effect was taken by Boyer as affirmation of the existence of the ZPE. But if the measured force were caused by electrostatic charge, the Casimir effect and the inferred existence of the ZPE would be contradicted and not affirmed.

Since Casimir, experiments performed at ambient temperature have generally confirmed the Casimir force at gaps less than about 0.25 μm, and since the tests were performed absent pressure in a vacuum, it can only be concluded that the Casimir force has a thermal origin. Conversely, a vacuum origin to the Casimir force would be indicated, if the Casimir force were confirmed in tests in a vacuum near absolute zero. But cryogenic [5] tests have never been performed, and therefore claims that the Casimir effect produces a force from the 'nothing' of the vacuum are speculative, and at the very least violate the conservation of EM energy.

In MEMS, a doubly clamped gold coated beam [7] was found to cause permanent adhesion. But the Casimir induced contact pressure was estimated [5] to be very small, about 1 bar at 10 nm. Certainly, low contact pressure between neutral and otherwise chemically non-reactive surfaces is unlikely to cause permanent adhesion, and therefore it may be concluded that electric charge is somehow produced in otherwise neutral bodies separated by microscopic gaps, the permanent adhesion caused by ESD.

Moreover, the Casimir effect does not explain the ESD observed [8] from otherwise neutral semiconductor materials in photolithography. Indeed, ESD is a common problem in 0.25 μm and below production lines. Why ESD is significantly enhanced in photolithography for gaps $\delta < 0.25\ \mu m$ is not fully understood or explained.

What this means is the standard Casimir theory for neutral bodies needs to be modified One such modification [9] of Casimir theory is based on VUV radiation produced by cavity QED induced EM radiation that liberates electrons from the plate surfaces and depending on differences in electron yield charges the plates by the photoelectric effect, the plate losing more electrons charging positive and the plate gaining electrons charging negative. Thus, the plates are attracted to each other, but by electrostatics – not the Casimir effect.

2. Purpose

Consistent with experiments that suggest the Casimir force is electrostatic and not neutral, the purpose of this paper is to show the neutral force as originally envisioned by Casimir does not exist.

3. Theories

In the derivation of the Casimir force, consider a pair of flat plates emitting BB radiation separated by an evacuated gap of dimension δ supporting a standing VUV wave as depicted in Fig. 1.

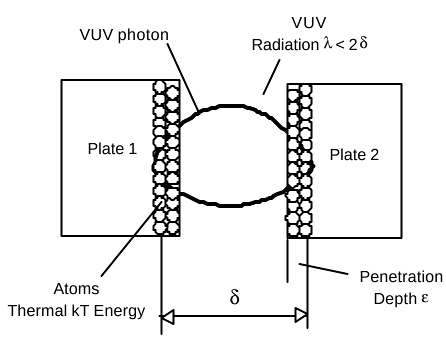

Fig. 1 Casimir Force Theories

3.1 Casimir

Historically, the derivation [1,10] of Casimir force F_c relies on Planck's formulation [2] of BB radiation that included the ZPE. The EM energy ΔU difference across either plate comprises suppressed U_{sup}, cavity U_{cav} and free space U_{free} contributions,

$$\Delta U = U_{sup} + U_{cav} - U_{free} \tag{3}$$

Unaware of cavity QED induced EM radiation, Casimir set the suppressed energy $U_{sup} = 0$.

$$\Delta U = U_{cav} - U_{free} = \frac{hc}{2\delta}\left[\sum_{\upsilon=1}^{\infty}\upsilon - \int_0^{\infty}\upsilon d\upsilon\right] = -\frac{1}{24}\frac{hc}{\delta} \tag{4}$$

Thus, the Casimir force F_c is attractive,

$$F_c = -\frac{\partial \Delta U}{\partial \delta} = -\frac{1}{24}\frac{hc}{\delta^2} \tag{5}$$

3.2 Modified Casimir

Einstein's formulation [3] of the BB radiation law is absent the ZPE. EM energy is present because the plates at temperature T emit BB radiation to the cavity. Similarity is found with the Casimir theory in that the EM energy difference ΔU across the plate,

$$\Delta U = U_{sup} + U_{cav} - U_{free} \tag{6}$$

Unlike the Casimir theory, the suppressed U_{sup} energy is not zero, but is conserved with resonant U_{res} energy in the gap. Thus, the BB energy in the cavity U_{cav} is increased by the resonant U_{res} energy,

$$U_{cav} = U_{BB} + U_{res} \tag{7}$$

where,

$$U_{BB} = \int_{c/2\delta}^{\infty} G d\upsilon$$

$$G = \frac{8\pi h}{c^3}\frac{\upsilon^3}{\left[\exp(h\upsilon/kT) - 1\right]}$$

$$U_{res} = N_{res}\frac{hc}{2\delta}$$

where, N_{res} is the number of resonant photons in the gap per unit volume.

$$N_{res} = \frac{8\pi}{c^3}\int \upsilon^2 d\upsilon = \frac{8\pi\upsilon^3}{3c^3} \sim \frac{8\upsilon^3}{c^3} \tag{8}$$

For $\upsilon = c/2\delta$, $N_{res} \sim \delta^{-3}$,

$$U_{res} = \frac{1}{2}\frac{hc}{\delta^4} \tag{9}$$

Writing the ΔU energy difference,

$$\Delta U = U_{cav} - U_{free} \tag{10}$$

where,

$$U_{cav} = \int_{c/2\delta}^{\infty} G d\upsilon + \frac{1}{2}\frac{hc}{\delta^4} \quad \text{and} \quad U_{free} = \int_0^{\infty} G d\upsilon$$

From Eqn. 7,

$$\Delta U = \frac{1}{2}\frac{hc}{\delta^4} + \left\{ \int_{c/2\delta}^{\infty} - \int_0^{\infty} \right\} \{G d\upsilon\} \qquad (11)$$

But,

$$U_{res} = \frac{1}{2}\frac{hc}{\delta^4} = \int_0^{c/2\delta} G d\upsilon \qquad (12)$$

Thus,

$$\Delta U = \left\{ \int_0^{c/2\delta} + \int_{c/2\delta}^{\infty} - \int_0^{\infty} \right\} \{G d\upsilon\} \qquad (13)$$

Clearly, ΔU is independent of δ. The modified Casimir force F_{mc} is,

$$F_{mc} = -\frac{\partial \Delta U}{\partial \delta} = 0 \qquad (14)$$

Thus, the Casimir force F_{mc} derived on the basis of the conservation of EM energy does not exist.

Alternative Proof

An alternative, but physical argument that the Casimir force does not exist is to show the EM resonant energy U_{res} of cavity QED induced EM radiation in the gap is the same as the EM energy U_{free} on the free surface, and therefore the Casimir force cannot exist. From Eqn. 6, it must be shown,

$$U_{sup} = \int_{c/2\delta}^{\infty} G d\upsilon \sim 0 \qquad (15)$$

Typically, the fractional f power emitted in the frequency interval ($c/2\delta$ - ∞) is equivalent to the wavelength interval (2δ - 0), or (0 - λ) where $\lambda = 2\delta$. The fraction f is determined [11] by expressing Eqn. 13 as the ratio of $W_{b\,(0-\lambda)}$ to the total power W_b,

$$f = \frac{W_{b(0-\lambda)}}{W_b} = \frac{W_{b(0-\lambda)}}{\sigma T^4} \qquad (16)$$

where, σ is the Stefan-Boltzmann constant and T is the absolute temperature. Usually, fractional power f is given in a numerical tabulation in terms of the product λT, e.g., Table A.14 of [11]. Typically, Casimir experiments have gaps $\delta < 0.25$ microns, or wavelengths $\lambda = 2\delta < 0.5$ microns. At ambient temperature, $T \sim 300$ K giving $\lambda T \sim 150$ micron-K. Thus, $f \sim 1.7\times10^{-5}$ and $U_{sup} \sim 0$, and therefore $U_{gap} \sim U_{free}$, so

$$\Delta U = U_{gap} - U_{free} \sim 0 \qquad (17)$$

Thus, the Casimir force does not exist,

$$F_{mc} = -\frac{\partial \Delta U}{\partial \delta} = 0 \qquad (18)$$

What this means is all of the BB radiation at ambient temperature is converted to EM radiation at the resonant frequency of the gap, i.e., $U_{free} \sim U_{gap}$.

4. Application

The QED physics of spontaneous frequency up-conversion in gaps is nicely illustrated by thermophotovoltaic (TPV) devices that produce electrical current from near IR photocells separated by a gap from a heated surface emitting far IR radiation, the EM resonance of the gap selected to coincide with the peak responsivity of the near IR photocell.

The steady TPV photocell current I produced from the conversion of radiative heat Q,

$$I = \xi Q \qquad (19)$$

where, ξ is the responsivity of the photocell in units of amperes per watt. Typical TPV photocell responsivity of silicon and germanium are illustrated in Fig. 2.

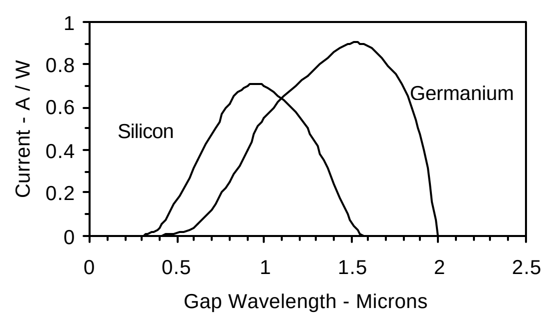

Fig. 2. Spectral Responsivity of Silicon and Germanium

The Stefan-Boltzmann law gives the classical radiative heat transfer *Q* between the heater and photocell,

$$Q = \sigma A \left[T_H^4 - T_{PC}^4 \right] \qquad (20)$$

where, σ is the Stefan-Boltzmann constant, A is the heater area, T_H and T_{PC} are the absolute heater and photocell temperatures.

However, the Stefan-Boltzmann law for thermal radiation is only valid [12] in large cavities having an EM resonance beyond the far IR. In microscopic gaps having dimension δ shorter than the half wavelength of far IR radiation, the Stefan-Boltzmann law is not valid, i.e., $\delta < 50$ µm.

Cavity QED induced heat flow Q based on far to near IR frequency up-conversion is proposed for radiative heat flow in gaps $\delta < 50$ µm. EM energy in the form of resonant gap photons at frequency $\upsilon = c / 2\delta$ having Planck energy $E = h\upsilon$ transfers thermal kT energy between heater temperature T_H and photocell at temperature T_{PC}. The heat flow Q,

$$Q = A\left(\frac{1}{\delta^2}\right)\left(\frac{1}{\tau}\right)mk\left(T_H - T_C\right) \qquad (21)$$

where, δ^{-2} is the number of standing near IR photons per unit heat flow area, m is the number of atoms transferring kT energy equivalent to the resonant photon $hc / 2\delta$, and τ is the recovery time for heat Q from the ambient to flow into the surface atoms.

For a circular heater chip [13] having R ~ 1 mm, the area $A = 3.1\text{x}10^{-6}$ m^2. Germanium has a peak spectral responsivity $\xi \sim 0.9$ A/W and occurs at λ_g = 1.55 µm and E_g = 0.80 eV. For temperatures T_H = 348 K and T_{PC} = 300K, and m = 29 average atoms, the cavity QED induced heat flow at heater temperatures T_H of 348, 378, and 408 K with τ = 260 to 340 ps in relation to experimental data (Fig. 4 of Reference 13) is illustrated in Fig. 3.

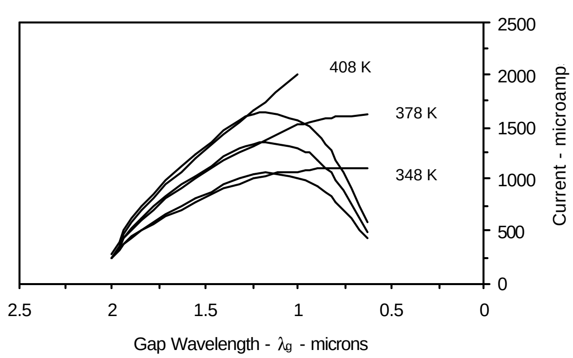

Fig. 3. Comparison of cavity QED Induced TPV Effect and Experimental Data

The comparison of the cavity QED induced TPV effect and experimental data in Fig. 5 shows good agreement from λ = 2 to 1.7 µm. But for λ = 1.7 to 0.6 µm, the agreement is poor. The experimental data suggests the response in the visible region is greater than that in the near IR which is inconsistent with the responsivity for germanium shown in Fig. 2.

The reason for the disparity with the cavity QED theory is the provision [13] of silicon dioxide spacers (~ 1 µm high) on the heater surface to control gap spacing.

Away from the spacers the heater chip deforms with the capacitance increasing as the gap decreases. Over this deformed area, the photocell current decreases because the resonant wavelength λ moves toward the visible away from the peak responsivity of the near IR photocell. Even so, the photocell continues to produce a net current because near the spacers, the gap remains at $\delta < 1$ µm having resonant wavelength $\lambda < 2$ µm that is in the near IR range of the responsivity of germanium. Indeed, Fig. 3 suggests the spacer height is closer to $\delta \sim 0.85$ µm because the deviation between QED theory and experiment begins at $\lambda \sim 1.7$ µm, after which the current tends to remain almost constant as the capacitance increases.

That the spacers are the cause of the disparity may be easily confirmed by reducing their height and should approach the current response predicted by cavity QED induced TPV effect.

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