Bubble dynamics under ultrasound

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Abstract Bubble nucleation under ultrasound is generally thought to occur by the opening of voids between liquid molecules during the expansion phase of the acoustic cycle. But this is only possible for ideal liquids with zero surface tension. In real liquids, bubble nucleation is precluded because the voids collapse by surface tension. For vapor bubbles to nucleate under ultrasound, a micron sized spherical particle of liquid molecules separated from the bubble wall by an evacuated annular gap is proposed to form in the liquid continuum, the particle radius given by the surface tension and ambient pressure. Bubble growth then proceeds from the expansion of the annular gap only to return to the continuum as the bubble wall collapses back on the particle.

1. Introduction and background

Traditionally, the nucleation of bubbles in water under ultrasound excludes the formation of a microparticle of liquid molecules formed by surface tension. Spherical bubble dynamics [1] was first derived by Rayleigh in 1917 and extended by Plesset to cavitation bubbles in 1949. Neglecting viscosity, the Rayleigh-Plesset (R-P) equation absent the particle,

$$P_{drive} = \rho\left[R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2\right] \qquad (1)$$

where,

$$P_{drive} = P_g + P_v + P_a \sin(2\pi f t + \phi) - 2S/R - P_{atm} \qquad (1.1)$$

P_{drive} is the pressure acting on the bubble wall of radius R, t is time, and ρ is the density of the liquid. The pressure P_{drive} includes gas pressure P_g, vapor pressure P_v of the liquid, atmospheric pressure P_{atm}, surface tension S, and the acoustic pressure P_a amplitude at frequency f and phase angle ϕ.

The history of R-P simulations of the bubble wall is replete with the problem of surface tension precluding bubble growth. Consider water at 20 C having surface tension S ~ 0.072 N/m. If the pressure $P_g + P_v$ at least compensates for atmospheric pressure P_{atm}, the bubble radius $R_o = 2S/P_{atm} \sim 1.44$ μm. For a typical acoustic pressure amplitude $P_a \sim 1.2\ P_{atm}$, Eqn (1.1) shows $P_{drive} = 0.2\ P_{atm} > 0$ and the bubble grows. But in ultrasound, only vapor bubbles may be nucleated, and therefore the gas pressure $P_g = 0$. At ambient temperature, the water vapor pressure $P_v << P_{atm}$, and therefore the bubble collapses because $P_{drive} \sim -0.8P_{atm} < 0$.

In the alternative, consider bubble nucleation in water under ultrasound to produce a particle [2] of diameter $2R_o$ as illustrated in Fig. 1.

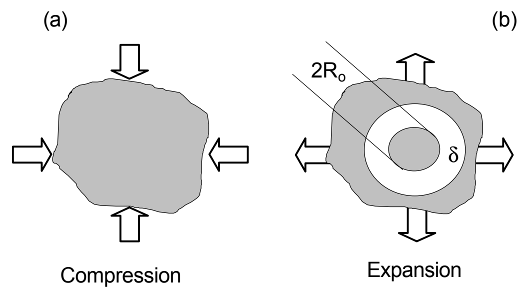

Fig. 1 Proposed bubble nucleation mechansim

Fig. 1(a) shows a region of liquid water during the hydrostatic compression phase of the acoustic wave. Fig. 1(b) shows the hydrostatic tension by the expansion phase of the wave causing the bubble wall to separate from the particle.

Initially, the annular space δ is infinitesimally small, but increases as the bubble expands, i.e., $R = R_o + \delta$. The bubble grows to maximum radius R_{max} and then collapses back on the particle, and therefore the minimum bubble radius $R_{min} = R_o$. Each ultrasonic cycle begins with the formation of the particle and surrounding annular evacuated gap followed by bubble growth in the continuum. Collapse returns the bubble wall to the particle to once again form the continuum for the beginning of the next cycle.

The proposed liquid particle under ultrasound is similar to the bubble nucleation [3] from solid particles of radius R_o. Indeed, the surface tension S causes the liquid pressure to become appreciably negative before the bubble starts growing at R_o. At nucleation, the initial radial velocity dR/dt at R_o over a wide range produced insignificant changes in the overall bubble response and could be ignored. To wit, the maximum bubble radius R_{max} and the collapse velocity V_c were found to be virtually unaffected by the surface tension perturbation at the beginning of the R-P simulation.

By beginning the R-P simulation of bubble dynamics at the radius R_o of the liquid particle, the bubble can only grow. Similarly, ending the R-P simulation by the bubble wall collapsing on the particle avoids the singularity of focussing the collapse energy over a zero area.

In the literature, the formation of the particle of radius R_o in bubble nucleation was cited [2] in relation to R_{min} during ultrasonic vibration, although the particle itself was not explicitly included in that R-P simulation.

Absent ultrasound, the particle forms anytime a liquid is abruptly perturbed to a state of hydrostatic tension. In this regard, the liquid particle is proposed formed as bubbles nucleate in the expansion [4] of supercooled water in the updraft of a thundercloud and in low pressure adjacent to obstructions in pipe flow [5].

2. Problem and resolution

The traditional R-P equation for the bubble wall explicitly includes surface tension at the instant of bubble nucleation. But because the liquid particle is absent the bubble, a zero surface tension is implicitly assumed for the particle. This is a problem - the bubble wall cannot have surface tension while at the same time the particle has zero surface tension.

This problem is resolved if the vapor bubbles under ultrasound nucleate from liquid particles formed by surface tension.

3. Analysis

Writing the R-P equation for *both* the particle and the bubble wall,

Particle

$$P_{drive} = -P_v - P_{atm} + 2S/R \sim 0 \qquad (2)$$

Bubble wall

At the instant of bubble nucleation,

$$P_{drive} = P_v + P_a \sin(2\pi ft + \phi) - 2S/R \qquad (2.1)$$

After bubble nucleation,

$$P_{drive} = P_v + P_a \sin(2\pi ft + \phi) - P_{atm} \qquad (2.2)$$

For the particle, $P_{drive} \sim 0$ because the particle is essentially in static equilibrium. But the expression for P_{drive} for the bubble wall at the instant of nucleation differs from that after nucleation.

At the instant of nucleation, the acoustic pressure amplitude P_a need only overcome the surface tension pressure $2S/R = P_{atm}$ because atmospheric pressure P_{atm} acts on both particle and bubble wall surfaces across the interface. Eqn (2.1) requires $P_a + P_v > 2S/R$.

Once the particle is formed, the surface tension in the R-P equation for the bubble wall may be neglected. This is justified because at a particle radius R_o of a few microns, the surface tension pressure 2S/R on the bubble wall rapidly vanishes by an increase of bubble wall radius R. Indeed, the impulse imparted to the bubble wall in the abrupt particle separation is likely sufficient to increase the radius of the bubble wall a few microns, thereby reducing the surface tension pressure to a negligible level. In the manner of [3], the surface tension perturbation following particle separation does not affect the overall response of the bubble wall. Thus, Eqn (2.2) requires $P_a + P_v > P_{atm}$.

3.1 Procedure

The R-P equation is solved for bubble nucleation in water at acoustic pressure amplitudes $P_a = 1.2P_{amb}$ at f = 26,500 Hz. Usually, the R-P simulation [6] initially assumes a gas filled bubble of air or argon having radius $R_o \sim 5$ μm.

In contrast, the R-P simulation here assumes bubble nucleation begins by the formation of a liquid particle having radius R_o at atmospheric pressure P_{atm}. Once the particle is formed, the surface tension is neglected in the R-P equation for the bubble wall.

Otherwise, the procedure [2] is followed. Breakup of the bubble occurs if the pressure P_B just beneath the surface of the bubble wall exceeds the water vapor pressure P_V,

$$P_B = -P_{drive} + \rho\left[R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2\right] > P_V \qquad (3)$$

If so, the R-P equation is no longer valid, and the 1-D Bernoulli equation [2] is used. The bubble radius R* designates breakup.

The vapor pressure of water remains near ambient throughout the bubble collapse, and therefore the use of the perfect gas law is justified. The particle volume reduces the bubble volume, but the particle surface area increases the area for condensation.

The Hertz-Knudsen relation for condensation is assumed. Condensation coefficients $\alpha < 1$ are cited [6] in the literature. But α is the probability of a vapor molecule sticking to a surface, and therefore it is unequivocal that in a bubble $\alpha = 1$ because it is certain that the water molecule does not escape the bubble wall. Hence, $\alpha = 0.4 < 1$ are not valid for bubbles.

3.2 Solutions

The R-P solution for bubble radius R and velocity V_C showed virtually the same response for condensation coefficients $\alpha = 0.4$ and 1 as shown in Figs. 2 and 3. The maximum radius $R_{max} \sim 30$ μm. Breakup of the bubble wall is found to occur at R* ~ 13 μm. In Fig. 3, the collapse velocity V_C is plotted from right to left beginning at the maximum R_{max} radius and ending at R_o. Maximum $V_C \sim 100$ m/s occurs at $R_o \sim 1.44$ μm.

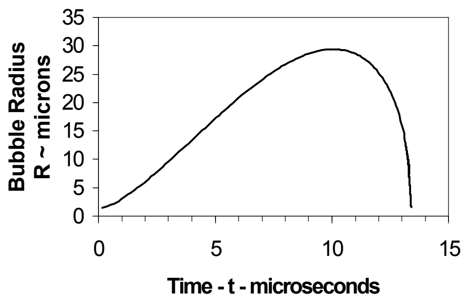

Fig. 2 Bubble R – response for $P_a \sim 1.2\ P_{atm}$

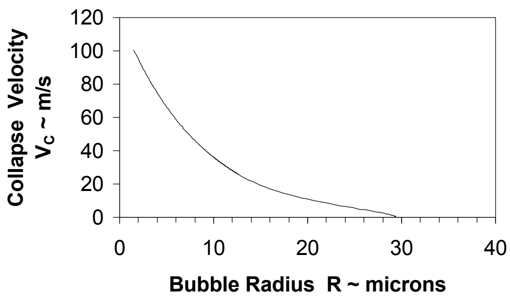

Fig. 3 Collapse velocity V_C

The water vapor pressure ratio P_v/P_{vo} during bubble collapse is depicted in Fig. 4. For $\alpha = 1$ and 0.4, the maximum $P_v/P_{vo} \sim 1.5$ and 9.6, respectively. The dip in the $\alpha = 1$ solution is caused by an increase in the condensation area by the particle and is only significant as the bubble wall area approaches that of the particle.

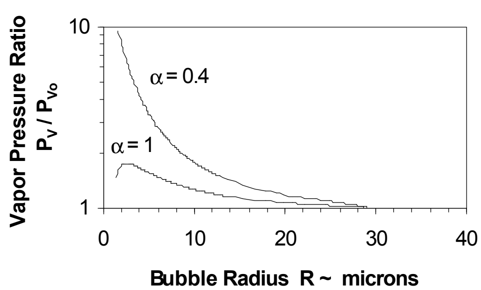

Fig. 4 Water vapor pressure ratio P_v/P_{vo}

4. Summary

Under ultrasound, surface tension precludes the nucleation of vapor bubbles from evacuated voids in the interstice between liquid molecules. Thus, the vapor bubbles observed under ultrasound are nucleated by some other mechanism. It is proposed that bubble nucleation finds origin in micron sized liquid particles formed by surface tension in the expansion phase of the acoustic wave. The R-P simulations show collapse of vapor bubbles under ultrasound proceed almost isothermally with little, if any increase in temperature, although there are very high pressures as the bubble wall collides with the particle. Since $V_C \sim 100$ m/s, the particle is subjected to a stagnation pressure $P_{stag} \sim 3 \times \frac{1}{2}\rho V_C^2 \sim 150$ bar every acoustic cycle.

Bubble collapse is not strictly isothermal, but the temperature increase of the water vapor by compression heating as the bubble volume decreases during collapse is insignificant. For water at 20 C, the vapor pressure $P_{vo} \sim 2.34$ kPa. Thus, the ratio $P_v / P_{vo} \sim 1.5$ for $\alpha = 1$ gives $P_v \sim 3.51$ kPa. The temperature is upper bound at equilibrium temperature of 27 C for a temperature increase of 7 C. For the hypothetical $\alpha = 0.4$ condensation coefficient, $P_v / P_{vo} \sim 9.6$ corresponds to vapor pressure $P_v \sim 22.5$ kPa and gives a temperature of 63 C for a temperature increase of 43 C.

5. Conclusions

Bubbles under ultrasound are vapor bubbles that nucleate from micron sized liquid particles formed by surface tension at ambient pressure.

Only in the hypothetical case of zero surface tension may the bubbles nucleate under ultrasound without forming a liquid particle.

Although for different reasons, the R-P simulations are consistent with Rayleigh's assumption in 1917 that the bubbles collapse isothermally at high pressure.

The R-P simulations are also consistent with Le Chatelier's principle known for over 100 years. To wit, in a liquid-vapor system, an increase in pressure caused by a decrease in volume will cause the system to retreat from the increased pressure by the conversion of some vapor into liquid without an increase in temperature.

References

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