1/f Noise by Quantum Mechanics

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Abstract. The 1/f noise commonly observed in EM measurements of power in circuit elements - capacitors, resistors, and inductors - is perhaps one of the oldest puzzles of contemporary physics. EM stands for electromagnetic. Simulations of 1/f noise made by a summation of Lorentzians for step changes of power in time, or by a Markov chain of such steps in a stochastic stationary process do indeed produce 1/f noise in the frequency domain, but lack a physical basis. The dramatic increase in 1/f noise in nanoelectronic circuit elements, and nanowires in particular suggest a physical basis may be found in the size effect of QM and even extended to submicron regions of ordinary circuit elements. QM stands for quantum mechanics. In nanoelectronics, QM precludes atoms from the heat capacity necessary to conserve Joule heat by an increase in temperature. Instead, conservation proceeds by the creation of charge that produces a step change of power in time, the Fourier transform of which gives the 1/f noise spectrum in the frequency domain.

Keywords: 1/f noise, nanoelectronics, nanowires quantum mechanics, Fourier transforms

1. Introduction

1/f noise is commonly observed in EM measurements of electronic circuits comprising capacitors, resistors, and inductors. Other processes in the natural world also display this behavior including stock prices and music suggest a generic origin might exist, but no generally recognized physical explanation of 1/f noise has been proposed. Indeed, the ubiquity of 1/f noise in diverse areas is one of the oldest puzzles of contemporary physics.

In this paper, the focus of 1/f noise is in EM measurements of nanoelectronics in general and nanowires in particular. Research in 1/f noise began in 1925 with Johnson and Nyquist who found [1,2] vacuum tubes emitted white noise. White noise has a constant power spectrum generated by the thermal agitation of charges inside a resistor by itself without any applied voltage. In contrast, 1/f noise sometimes called pink noise differs in that under applied voltage the frequency f of the power spectrum is not constant, but rather increases as f is lowered, i.e., having a slope of -1 on a log-log plot of noise vs. frequency.

Over the past 80 years, explanations of 1/f noise have proved elusive. Schottky [3] proposed a mathematical model of exponentially decaying current pulse I caused by the release of electrons from the cathode of a vacuum tube, i.e., $I = I_o \cdot e^{-bt}$, where I_o is the initial step, b is the relaxation rate and t is time. For a train of such pulses at an average rate, the power spectrum is of the Lorentzian form which is constant near f = 0 and for large f is proportional to $1/f^2$ with a narrow transition region where the power spectrum resembles that of 1/f noise. Later, Bernamont [4] pointed out only a superposition of such processes with a variety of relaxation rates yield 1/f noise for a reasonable range of frequencies.

Recently, Erland et al. [5] have proposed that 1/f noise is a stationary stochastic process having a mean and variance that do not change over time, and therefore may be modeled as a Markov chain in which one observation depends stochastically only on the immediately previous one.

In the literature, studies generally avoid the physical mechanism that underlies 1/f noise with emphasis placed on mathematical explanations of how it may occur.

But what is the physical mechanism that underlies 1/f noise?

Today, 1/f noise is generally explained by free electrons in the Hooge [6] relation,

$$\frac{S_R}{R^2} = \left(\frac{S_I}{I^2}\right)_V = \frac{\alpha}{fN} \tag{1}$$

where, S_R and S_I are the spectral power density of resistance R and current I, f is the frequency at which the noise is measured, N is the total number of free electrons, and α is an empirical dimensionless constant.

However, Hooge [7] described the difficulty of reconciling LFN with the residence time of the electron in the size of the sample. LFN stands for low frequency noise. If 1/f noise is a summation [1-5] of Lorentzians having long characteristic times t >> 1s, the electrons must stay in the sample much longer than a few seconds. In fact, Weissman [8] argued in a sample with a length of 1 cm, an electron only stays about 0.1 s as its diffusion coefficient D is of order 0.001m²/s. Weissman asked Hooge:

How can electrons stay in the sample and produce LFN at frequencies below 1 Hz?

Hooge's answer [7] was the 1/f noise is caused by lattice scattering of electrons. However, this is unlikely because phonon frequencies are far higher than LFN. What this means therefore is the number N of free electrons in (1) has nothing to do with 1/f noise.

To this day, 1/f noise continues to be described by the Hooge relation for scattering of electrons. An exception was Handel [9] who proposed a quantum 1/f theory that claimed LFN is caused by low-frequency IR photons that are absorbed or emitted in each scattering event. However, the half-wavelength of LFN is far longer than the dimensions of the samples, and therefore the cage effect, better known today as QED cannot be invoked as the EM confinement mechanism by which the EM energy in scattering of electrons produces 1/f noise in the IR. QED stands for quantum electrodynamics. In support of Handel's 1/f theory, van Vliet's [10] claim that the absence of an EM confinement mechanism to create the IR photons is not a serious problem must be rejected. Without EM confinement, the IR photons are not created. It suffices to say the experimental results cited by Hooge [6,7] do not support Handel's 1/f theory.

2. Purpose

The purpose of this paper is to support the hypothesis that 1/f noise is a QM effect that produces charge as current passes through nanoelectronic circuit elements.

3. Theory

3.1 QM Restrictions

The QM restrictions of size in nanoelectronics depends on the EM confinement given by the Einstein-Hopf [11] relation for the Planck energy E of the atom as a harmonic oscillator,
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$$E = \frac{\frac{hc}{\lambda}}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \tag{2}$$

where, h is Planck's constant, c the speed of light, k Boltzmann's constant and T absolute temperature. Planck energy E is a measure of the capacity of the atom to absorb heat and is shown in terms of the EM confinement wavelength λ is shown in Fig. 1.
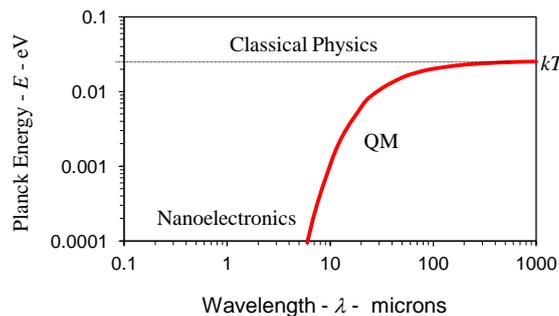

Fig. 1 Planck energy of the atom in Classical Physics and QM at 300 K

Fig. 1 shows QM differs from classical physics in that the size of nanoelectronic circuit elements precludes the atom from having heat capacity to conserve absorbed Joule heat by an increase in temperature.

3.2 TIR Confinement

Nanoelectroncs cannot conserve Joule heat by increasing in temperature. Contrary to van Vliet's [10] claim that EM confinement is not necessary to create IR photons, QED induced radiation requires TIR to create photons *inside* nanoelectronic circuit elements. TIR stands for total internal reflection,

TIR has a long history beginning with Tyndall in 1870 who observed if the refractive index of a body is greater than that of the surroundings, absorbed light is trapped at its surface. However, TIR in nanoelectronics has an important significance and need not be limited to light absorption. Unlike the macro world, the size of nanoelectronic elements corresponds to high surface to volume ratios, and therefore heat from any EM source (lasers, Joule heat, etc.) is absorbed almost totally in the element surface. Since the surface of the circuit element corresponds to its TIR wave function, QED induces the absorbed heat to undergo the spontaneous creation of QED photons *inside* the element. TIR confinement is not permanent, but rather sustains itself only during the absorption of EM energy, i.e., absent EM energy absorption, there is no TIR confinement and QED radiation is not created.

Similar to creating QED photons of wavelength λ by supplying EM energy to a QM box with sides separated by $\lambda/2$, the absorbed EM energy is frequency up-converted to the characteristic dimension D_C of the circuit element, e.g., for a nanowire having refractive index n and diameter d, the QED photon energy E and frequency υ are:

$$E = h\upsilon, \quad \upsilon = \frac{c}{\lambda}, \quad \lambda = 2nd \tag{3}$$

3.3 QED Photons and Rate

QED photons created by conserving Joule heat produce excitons by the photoelectric effect. However, only a fraction η of the power P produces photons, the remainder $(1-\eta)$ is lost to the surroundings, i.e., the fraction η of Joule heat I^2R is conserved by creating number N_P of QED photons *inside* the element,

$$\frac{dN_P}{dt} = \frac{\eta}{E}P = \frac{\eta}{E}I^2R \quad \text{and} \quad \frac{dN_{ex}}{dt} = Y\frac{dN_P}{dt} = \frac{\eta YI^2R}{E} \tag{4}$$

where, I is current, R resistance, and dN_p/dt is the rate of QED photons created. By Einstein's photoelectric effect, dN_{ex}/dt is the rate of excitons produced and Y is the exciton yield/ QED photon. Unlike the probabilistic creation of excitons from an external source of radiation having $Y \ll 1$, QED photons created *inside* the circuit element by the conservation of EM energy have $Y = 1$.

3.4 QM Charging

Excitons form in proportion to the fraction ηP of QED photons created. The Q_H and Q_E are the number of charged carriers of holes and electrons. For the circuit element of length L, the high electric field F causes the charge carriers to separate and by their respective μ_E and μ_H mobility rapidly move to opposite polarity terminals. The rate $\eta YP/E$ of excitons created is balanced by the hole Q_H and electron Q_E charges lost by neutralization at the terminals.

$$\frac{dQ_E}{dt} = \frac{\eta YI^2R}{E} - Q_E\frac{\mu_E F}{L} \quad \text{and} \quad \frac{dQ_H}{dt} = \frac{\eta YI^2R}{E} - Q_H\frac{\mu_H F}{L} \tag{5}$$

Both exciton (hole and electron) equations are identical allowing the hole response to represent that of the electron. For a constant voltage V_o across the circuit element, the field $F = V_o/L$,

$$\frac{dQ_H}{dt} = \frac{\eta YI^2R}{E} - \frac{\mu_H V_o}{L^2}Q_H \tag{6}$$

The hole Q_H solution is given by,

$$Q_H = \frac{L^2}{\mu_H V_o}\left\{\frac{\eta Y I^2 R}{E}\left[1 - \exp\left(-\frac{\mu_H V_o}{L^2}t\right)\right] + \frac{\mu_H V_o}{L^2}Q_{H0}\exp\left(-\frac{\mu_H V_o}{L^2}t\right)\right\} \tag{7}$$

On average, the holes and electrons are centered in the circuit element need to move L/2 to reach the voltage terminals, the resistance R and current I are,

$$R = \rho\frac{L}{2A} = \frac{L}{2A}\frac{1}{e(\mu_E Q_{EO} + \mu_H Q_{HO})/AL} \approx \frac{L^2}{4e\mu_H Q_H} \quad \text{and} \quad I = \frac{V_o}{R} \tag{8}$$

The resistivity ρ assumes $\mu_E = \mu_H$ with the same number Q_E of electrons as the Q_H holes. Note ρ requires units of per unit volume, where volume is AL and A is the circuit element area. The initial resistance R_o gives the initial number Q_{HO} of holes,

$$Q_{HO} = \frac{L^2}{4e\mu_H R_O} \tag{9}$$

4. Application

QED theory is compared with experimental data [12] of 1/f noise for SnO_2 nanowires. The hole mobility $\mu_H = 0.0172$ m^2/V-s is assumed for SnO_2 10-50 nm wire diameters d with length L > 10 microns. The initial resistance Ro = 25x10^6 ohm is taken from (Fig. 1(a) of [12]) for V_o = 2.5 volts at a current I = 0.1 μA. The power P = I^2R ranged from 1 to 3 μW. The Planck energy E and rate dN_p/dt of QED photons created are shown in Fig. 2. For a 20 nm diameter wire, the exciton velocity and rise time, and initial Q_{HO} and final Q_H number of holes in the nanowire is given in Fig. 3 and 4, respectively.
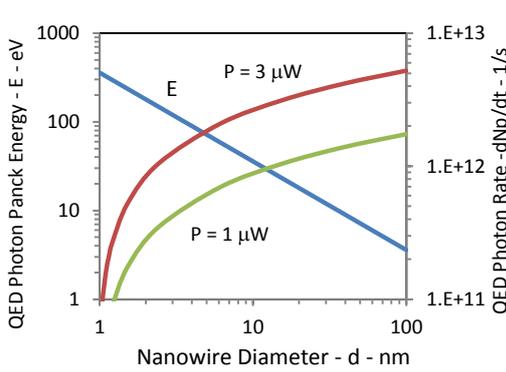

Fig. 2 QED Photon Energy and Creation Rate.
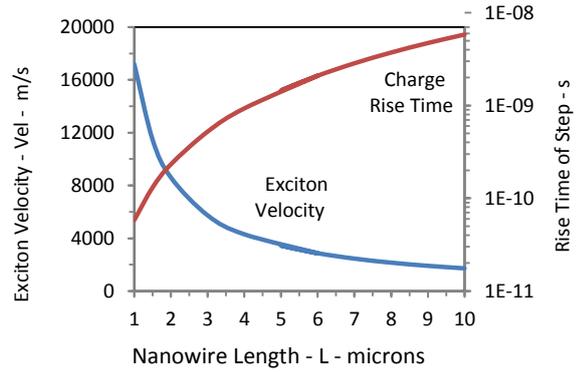Power = 1 and 3 μW
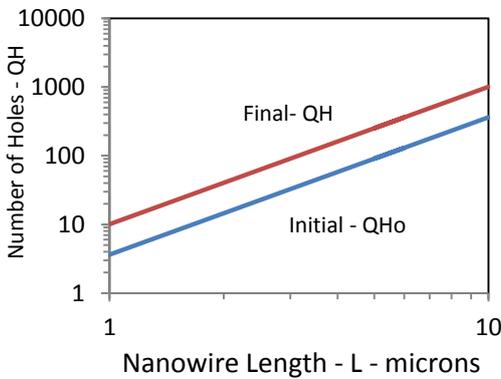

Fig. 3 Exciton velocity and Rise time

Fig. 4 Initial and final Number of Holes
in SnO$_2$ nanowires

The charge creation as the current enters the nanowire is abrupt, i.e., the time constant from (7) is less than 6 ns at lengths L < 10 microns. What this means is the step in charge may be treated as a step function in power.

The Fourier transform (FT) of a unit step change of power in the time domain gives the 1/f spectrum in the frequency domain,

$$G(2\pi f) = \int_{-\tau/2}^{\tau/2} 1e^{-j\omega t}\, dt = \frac{\sin(\pi\tau)}{\pi f} \tag{10}$$
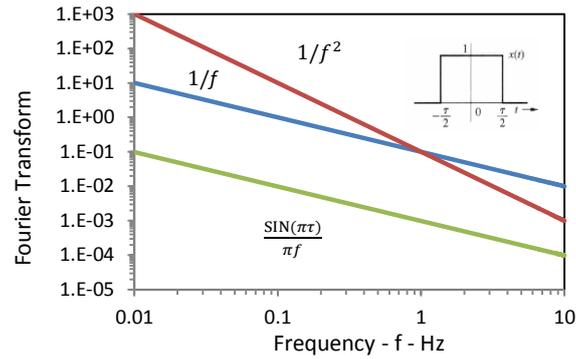
as shown to have the same slope as 1/f in Fig. 5.

Fig. 5 FT of step change in power is shown to have the same slope as the usual 1/f spectra found experimentally for LFN. The FT is based on the unit step shown in the inset. The $1/f^2$ spectrum is shown only for comparison.

5. Summary and Conclusions

Charge continuously created in nanowires by QM is consistent with a Markov chain of step changes in power in the manner of a stochastic stationary process. However, a Markov chain is not necessary as a continuous step change of power in time by QM naturally occurs upon passing current through nanowires, the FT of which gives the 1/f spectrum in the frequency domain.

The mechanism that underlies the 1/f spectra in nanoelectronics is the QED induced conservation of Joule heat by non-thermal EM radiation instead of an increase in temperature. Under a constant voltage across the nanowire, the EM radiation having high Planck energy creates excitons by the photoelectric effect, the holes of which abruptly lower the resistance to produce a step change of power in time.

1/f noise in SnO_2 nanowires has nothing to do with a large number of free electrons by Hooge's theory, but rather on the creation of small numbers < 50 holes created by the photoelectric effect from QED induced non-thermal EM radiation.

Imposing a step change of current in a nanowire is expected to increase the 1/f noise, thereby verifying the hypothesis based on QM that charge is produced upon passing current through a nanowire.

6. References

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