

Quantum Mechanics and Nanoelectronics

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Abstract— Electrical charge in nanoelectronic circuit elements – resistors, capacitors, and inductors - is explained by the quantum mechanics (QM) requirement that specific heat vanishes in nanostructures. In the memristor, Joule heating therefore cannot be conserved by an increase in temperature. Instead, conservation proceeds by the QED induced creation of photons within the TIR confinement frequency of the memristor. QED stands for quantum electrodynamics and TIR for total internal reflection. The TIR confinement of QED photons is enhanced by the fact the absorbed energy is concentrated in the TIR mode because of the high surface to volume ratio in nanostructures. The QED photons having Planck energy beyond the ultraviolet produce excitons, the electrons and holes of which reduce the initial resistance of the memristor, but the resistance is recovered later in the same cycle as the electrons and holes are attracted to and destroyed by the polarity of the voltage terminals. Earlier work on the numerical simulations of the HP memristor is updated and extensions of QED induced radiation to nanoscale heat transfer made to nanowire memristors, the Ovshinsky Effect, phase-change random access memory – PCRAM, nanocars, and ballistic transport in nanocontacts.

Keywords—memristor, nanoelectronics, heat transfer, PCRAM, quantum mechanics, quantum electrodynamics.

I. INTRODUCTION

ELECTRONICS became popularized in a paper [1] by Chua in 1971 that claimed a passive two-terminal circuit element existed having a resistance that depended on the time–integral of the current. Based on symmetry arguments alone the notion was held that in electronic circuitry based on the three common elements - the resistor, capacitor, and inductor was incomplete. For completeness, a fourth element [2] called a memristor was proposed. But lacking an actual prototype, the memristor lay dormant for almost 40 years until a group [3] at Hewlett-Packard (HP) in 2008 announced the development of a switching memristor based on a thin film of titanium dioxide (TiO₂) sandwiched between platinum (Pt) electrodes.

The HP memristor has a linear relation of resistance to charge provided the current stays within limits [4]. In the OFF state, the memristor has the usual resistance. But oxygen vacancies in the TiO₂ are assumed, the vacancies acting as positive charges that reduce resistance in the ON state. Between OFF and ON states, current flow under positive bias causes the electrons to move to the positive terminal and positive charged vacancies to move toward the negative terminal; while for negative bias, the charges reverse directions. If the bias voltage is set to zero, the current vanishes and the memristor resistance retains the resistance it had at when the current stopped.

The HP memristor [3] is a variable resistor dependent on the amount of charge Q transferred. The voltage V across the memristor terminals is,

$$V = IM(Q) \quad (1)$$

where, I is the current and $M(Q)$ the resistance. The relation of the current I to the charge Q is,

$$I = \frac{dQ}{dt} \rightarrow Q = \int Idt \quad (2)$$

Charge Q is therefore the time integral of the current I . If $M(Q)$ does not change with charge Q , the resistance R given by Ohms law, i.e., $M(Q) = R = V/I$. Similarly, the power P dissipated by the memristor is,

$$P = IV = I^2M(Q) \quad (3)$$

Currently, HP memristor theory assumes positive charge from oxygen vacancies is the source of switching, but the theory is phenomenological lacking a physical basis to allow extensions to other memristors without vacancies. In fact, many experiments reported over the past 50 years show memristor behavior, e.g., sandwiched molecular layers [5] between gold electrodes, and modification [2] of electrical conduction in solid electrolytes, all of which exclude positive charge in vacancies. But sandwiched material between electrodes is not necessary. Indeed, memristor behavior is observed in a single material without electrodes, e.g., gold [6] and silicon [7] nanowires. Lacking vacancies, explanations of memristor behavior assume the presence of space charge, but the mechanism by which the space charge is produced is not identified. Space charge is also claimed [8] to explain light emission from organic memristors.

Recently, HP announced [9] a breakthrough in memristor research by identifying the location of oxygen vacancies to depend on the formation of the anatase phase from the normal amorphous phase of TiO₂. Since the formation of anatase from amorphous phase requires temperatures of 625 C, it was assumed that high temperatures in Joule heating occurred during switching. But the sample underwent electroforming prior to switching so it is not clear whether the phase change was caused by electroforming or memristor operation..

QM precludes temperature increases during switching provided submicron samples are sandwiched in materials of a lower refractive index, i.e., thick chalcogenide films do not follow QM and do indeed increase in temperature. Similarly, the melting of PCRAM films thought caused by high

temperatures from laser or current pulses [10] based on the Ovshinsky Effect [11] may be nothing more than creation of charge carriers by QED radiation.

QED theory argues space charge is created anytime EM energy is absorbed in isolated nanostructures, and need not be related to nanoelectronics. For instance, the movement of nanocars upon heating [12] was recently much publicized. Nanocars are molecular prototypes of actual cars having wheels on a pair of axles supported on a chassis. Upon laying down a number of nanocars at random on a flat gold surface, the nanocars were observed to move by heating the surface. Electrostatics was suspected, but classical physics cannot explain how charge is generated by heating.

Similarly, heat transfer in interconnects between nanoelectronic circuit elements is thought [13] to occur by ballistic transport by phonons creating temperatures across the nanocontact. Instead, QED theory suggests the temperature measurements were those of the macroscopic sample to which the nanocontacts were attached - not the nanocontacts.

Finally, memristors are submicron, and therefore QM is expected in any explanation of charging. The applicability of QM to memristors is supported by the fact that only at the nanoscale [3] is the memristor behavior detectable. Supramicron memristors behave just like ordinary resistors, where resistance is equal to the voltage divided by the current.

In this paper, the HP memristor is used as an illustrative example of the QED radiation produced in nanoelectronic elements. Prior QED theory [14] on QM in memristors is repeated for pedagogical reasons, although extensions are made to other nanostructures showing memristor behavior.

II. PURPOSE

Propose the charge, in nanoelectronic circuit elements is caused by the photoelectric effect from QED radiation created from the conservation of Joule heating that otherwise is conserved by an increase in temperature.

III. THEORY

Memristors are generally thin films and nanowires. Fig. 1 shows thin films having material of thickness d sandwiched between metal electrodes including a nanowire of a single material having diameter D and length L . Bias voltage is applied producing current I .

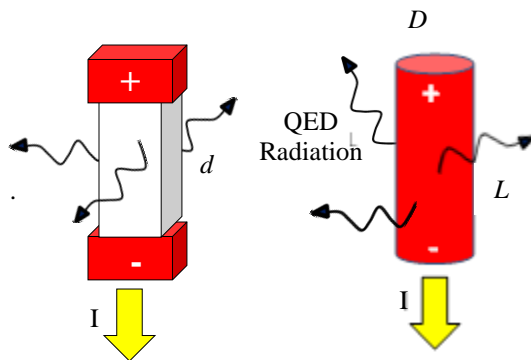


Fig. 1. Memristors – thin films and nanowires

A. QM Restrictions

To understand how QED radiation is produced in memristors, consider the QM restriction on heat capacity in conserving Joule heat by an increase in temperature. Unlike classical physics, the specific heat capacity of the atom by QM depends on its EM confinement. At 300 K, the Einstein-Hopf relation giving the average Planck energy for the harmonic oscillator in relation to kT energy (or heat capacity) and thermal wavelength $\lambda_T = hc/kT$ is shown in Fig. 2.

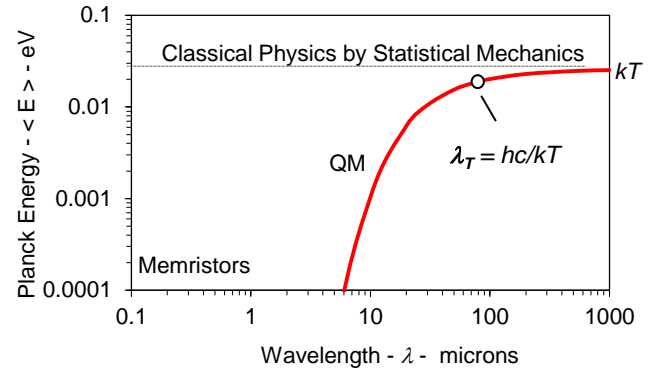


Fig. 2 Classical and QM Oscillators - Heat Capacity at 300 K

Unlike classical oscillators allowing the atom to have heat capacity at all wavelengths, QM oscillators only have heat capacity for $\lambda > \lambda_T$ and restrict heat capacity for $\lambda < \lambda_T$. At 300 K, $\lambda_T = 50$ microns. Fig. 2 shows the heat capacity is less than kT for $\lambda < \lambda_T$ and is only available for $\lambda > \lambda_T$. For memristors having $\lambda < 1$ micron, QM by requiring heat capacity to vanish precludes any increase in temperature upon Joule heating.

A. QED Confinement

Memristors lack heat capacity and cannot conserve absorbed EM energy by an increase in temperature. Instead, conservation may only proceed by the QED induced frequency up-conversion of the absorbed EM energy to the TIR confinement frequency of the memristor. Since memristors have high surface to volume ratios, the absorbed EM energy is confined by TIR almost entirely in the memristor surface. The TIR confinement is momentary and occurs only upon absorption of EM energy, and therefore the TIR confinement effectively sustains itself.

Similar to creating QED photons of wavelength λ by supplying EM energy to a QM box with sides separated by $\lambda/2$, the absorbed EM energy is frequency up-converted to dimension D_c . The QED photon energy E and frequency f are:

$$E = hf, \quad f = \frac{c}{\lambda}, \quad \lambda = 2nD_c \quad (4)$$

where, h is Planck's constant, c the velocity of light, and n the refractive index of the memristor. Thin films and nanowires have D_c of d and D , respectively.

C. QED Photon Energy and Rate

Classical heat transfer conserves absorbed EM energy by an increase in temperature, but is not applicable to memristors

because of QM restrictions on heat capacity. Instead, the EM dissipative power P is conserved by creating number N_P of QED photons inside the memristor having Planck energy E at the photon rate dN_P/dt ,

$$\frac{dN_P}{dt} = \frac{P}{E} = \frac{I^2 M(Q)}{E} \quad (5)$$

Only a fraction η of QED radiation creates excitons, the remainder $(1-\eta)$ is lost to the surroundings. By the photoelectric effect, the rate dN_{ex}/dt of excitons created, each exciton comprising an electron and hole is,

$$\frac{dN_{ex}}{dt} = \eta e \frac{dN_P}{dt} \quad (6)$$

where, e is the electron charge. The charge Q produced,

$$Q = \int \frac{dN_{ex}}{dt} dt = \eta e \int \frac{dN_P}{dt} dt \quad (7)$$

But the charge Q produced by QED radiation differs from that currently assumed and given in (2),

$$Q = \eta e \int \frac{dN_P}{dt} dt \neq \int I dt \quad (8)$$

D. QED Photons, Excitons, and Charge

The creation of charge Q in the memristor depends on the electrical dissipative power $P = I^2 M(Q)$. If current $I > 0$, QED photons are produced at rate $dN_P/dt > 0$, and therefore charge Q is produced. Note reversal of current does not alter the rate dN_P/dt of QED photons created and charge Q produced. But if current $I = 0$, QED photons are not created and no charge is produced.

Charge Q is produced from excitons in the memristor by QED photons having Planck energy greater than the band gap. The excitons comprising electron-hole pairs form as the QED photons leave a mobile positive charged hole corresponding to the charge Q .

The memristor resistance $M(Q)$ depends on the conductivity σ and resistivity ρ given by the number density of electrons N_E and holes N_H ,

$$\sigma = \frac{1}{\rho} = e(N_E \mu_E + N_H \mu_H) \quad (9)$$

where, μ_E and μ_H are the electron and hole mobility.

However, electron-hole pairs may recombine to create another photon having lower energy than the QED photon. But this is unlikely in memristors because the electrons separate from the holes under the high field F across the memristor, where V is the voltage drop and d is the material thickness. If the current stops, the hole charge Q_H is trapped. Upon the application of the bias voltage, the resistance $M(Q)$ begins with the value it had when the current stopped, i.e., the memristor remembers the last resistance.

E. QM Charging

The charging Q of the memristor occurs if $|I| > 0$. Excitons form in proportion to the fraction ηP of QED photons absorbed. Q_H and Q_E are the number of the positive charged holes and negative charged electrons created. Under the high electric field F , the holes and electrons promptly drift toward the opposite polarity voltage terminals as shown in Fig. 3.

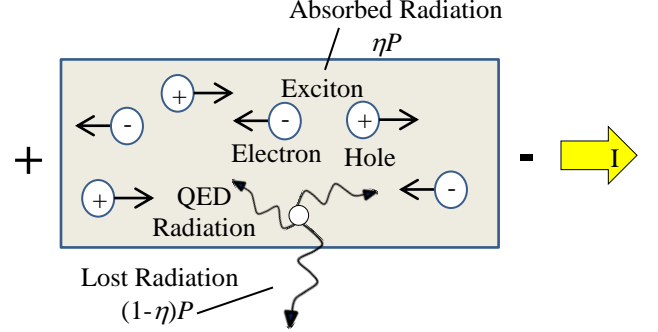


Fig. 3 Memristor - Charge Distribution – Positive Voltage Bias

The number of excitons $\eta P/E$ created is balanced by the electron Q_E and hole Q_H charges moving toward opposite polarity voltage terminals by their respective μ_E and μ_H mobility in the electric field F . Excluding the electrons in current I flowing through the memristor,

$$\frac{dQ_E}{dt} = \frac{\eta P}{E} - Q_E \frac{\mu_E F}{d} \quad (10)$$

$$\frac{dQ_H}{dt} = \frac{\eta P}{E} - Q_H \frac{\mu_H F}{d} \quad (11)$$

Both exciton electron and hole equations are identical allowing the hole response to represent the electron for the same mobility. Taking $F = V/d$ and $V = V_o \sin \omega t$ gives,

$$\frac{dQ_H}{dt} = \frac{\eta P}{E} - \frac{\mu_H V_o \sin \omega t}{d^2} Q_H \quad (12)$$

Equation (12) is solved with an integrating factor. The hole Q_H solution is given by,

$$Q_H \exp\left(-\frac{\mu_H V_o}{\omega d^2} \cos \omega t\right) = \frac{\eta P}{E} \int \exp\left(-\frac{\mu_H V_o}{\omega d^2} \cos \omega t\right) dt \quad (13)$$

where, the circular frequency $\omega = 2\pi f$ and f is frequency of the voltage. The electron Q_E solution is the same.

On average, the holes and electrons are centered in the film d and need to move $d/2$ to reach the voltage terminals, the resistance R is,

$$R = \rho \frac{d}{2A} = \frac{d}{2A} \frac{1}{e(\mu_E Q_{EO} + \mu_H Q_{HO})/Ad} \approx \frac{d^2}{4e\mu_H Q_H} \quad (14)$$

For simplicity, the resistivity ρ assumes $\mu_E = \mu_H$ with the same number Q_E of electrons as the number Q_H of holes. Note ρ requires units of per unit volume, where volume is Ad and A is

memristor area. The initial resistance R_o corresponds to the number Q_{HO} of hole charges,

$$Q_{HO} = \frac{d^2}{4e\mu_H R_o} \quad (15)$$

The current I is,

$$I = \frac{V}{R} = \frac{V_o \sin \omega t}{R} \quad (16)$$

IV. APPLICATIONS

A. HP Memristor

The HP memristor [3] is comprised of TiO_2 having thickness $d = 50$ nm sandwiched between 5 nm electrodes, one Pt and the other Ti. The area A is not known and is assumed to be $200 \times 200 \text{ nm}^2$.

1. *QED Photons and Rate* The power $P = I^2 R$ may be determined from $I_o = V_o/R_o$. Only fraction ηP is assumed absorbed and converted to QED photons. From (4), the QED photons have Planck energy $E = hc/2nd$. The refractive index n of TiO_2 for rutile and anatase, $n = 2.7$ and 2.55 , respectively. For TiO_2 thickness $d < 50$ nm, the Planck energies E are > 4.6 and 4.87 eV, respectively. Equation (5) gives the QED photon rates dN_p/dt as 1.36×10^{16} and $1.28 \times 10^{16} \text{ s}^{-1}$.

2. *Simulations* The bandgap of TiO_2 is about 3.2 eV, and therefore QED photons having Planck energies > 4.6 eV, produce excitons. In the simulations of HP data, the frequency $f = 5$ kHz and voltage $V_o = 1$ V was maintained. HP assumed (Fig. 2-3 of [3]) oxygen vacancies have mobility $\mu_H = 10^{-10} \text{ cm}^2/\text{V-s}$, but the simulations here could only be obtained over a mobility range from 10^{-8} to $10^{-6} \text{ cm}^2/\text{V-s}$.

Figs. 4 and 5 show the transient R and I with the corresponding I - V curve for $R_o = 100$ ohms, $P = 10$ mW, $\mu_H = 2 \times 10^{-6} \text{ cm}^2/\text{V-s}$, and $\eta = 10^{-8}$. The minimum resistance $R_{min} \sim 0.5$ ohms gives $R_o/R_{min} \sim 200$. For $\eta = 0$ the I - V curve is exactly linear with constant $R_o = 100$ ohms and $R_o/R_{min} = 1$. Figs. 6 and 7 gives the solution for $R_o = 250$ ohms, $P = 20$ mW, $\mu_H = 7 \times 10^{-8} \text{ cm}^2/\text{V-s}$, and $\eta = 10^{-8}$. The ratio $R_o/R_{min} \sim 83$ with $R_{min} \sim 3$ ohms

The I - V curves shows regions of negative dI/dV similar to HP data. The HP mobility is about 3-4 orders of magnitude lower than found here. High mobility is consistent with kHz frequencies. Nevertheless, the simulations are reasonable estimates of HP data. Similarity is found with mathematical [15] approximations to memristor behavior

The simulations show frequency f is related to mobility μ_H that may be understood by the time t^* for holes to move the distance d between terminals, i.e., $t^* = d^2/\mu_H V_o$. Setting t^* to half-period $1/2f$ gives $f = \mu_H V_o/2d^2$ thereby allowing an estimate of the frequency for a given mobility from which the solution may be tuned. For $d = 10$ nm and $\mu_H = 10^{-8} \text{ cm}^2/\text{V-s}$ is $10^{-12} \text{ m}^2/\text{V-s}$, the frequency $f = 5$ kHz.

Alternatively, if f is given, the computed value of μ_H may be considered a QED solution minimum. Below μ_H the solutions diverge. Increasing μ_H generates a family of I - V curve from linear to elliptical to the memristor shape shown in Figs. 5 and 7. Thereafter, the QED solutions diverge.

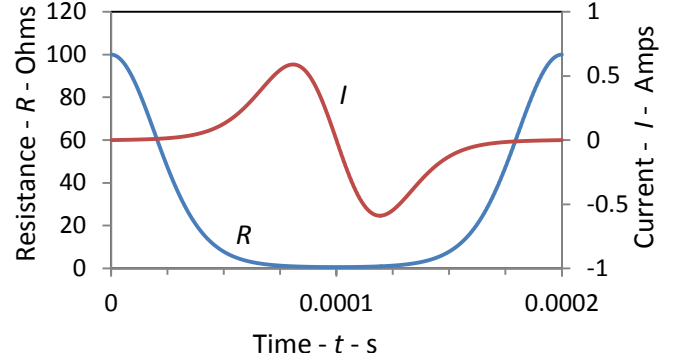


Fig. 4 Transient R and I - $d = 50$ nm

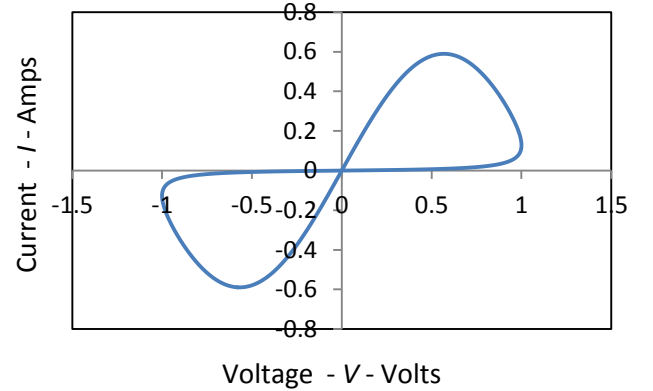


Fig. 5 I - V Curve - $d = 50$ nm, $\mu_H = 2 \times 10^{-6} \text{ cm}^2/\text{V-s}$

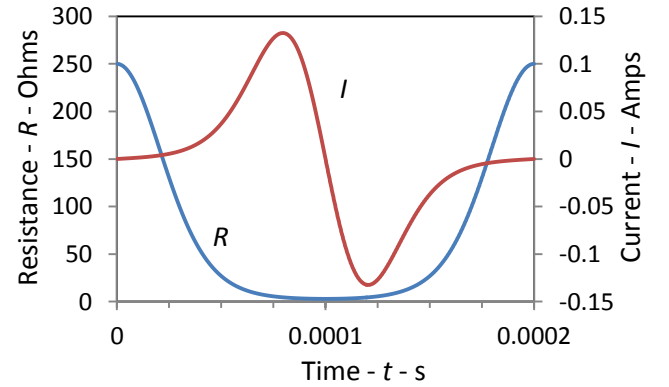


Fig. 6 Transient R and I - $d = 10$ nm

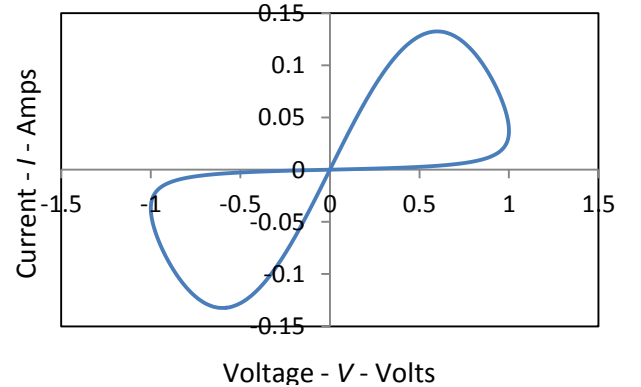


Fig. 7 I - V Curve - $d = 10$ nm, $\mu_H = 7 \times 10^{-8} \text{ cm}^2/\text{V-s}$

B. Other Memristors

1. *Gold Nanowires* Memristor behavior is found [6] in single component gold nanowires having rectangular shape: thickness $d = 20$ nm, width $W \sim 100$ nm, and length $L \sim 300$ nm. Only a single Au material was used in contrast to the HP memristor composed of TiO_2 and Pt. Oxygen in vacancies are excluded, yet the memristor behavior was nevertheless observed. Memristor behavior was explained by resistance changes caused by the thinning of the nanowire in electromigration.

In contrast, the QM of QED radiation allows the charge Q to be created and destroyed within the same switching cycle as shown above for the HP memristor. Indeed, the I - V curves for the Au nanowire in (Fig. 3 of [6]) are virtually identical to those derived here in Figs. 4 through 7. Explanations that temperature gradients along the wire are the source of the memristive effect are contrary to QM that precludes temperature changes at the nanoscale. Instead, space charge created by QED radiation is a more likely explanation for the memristive effect in Au nanowires.

2. *Silicon Nanowires* Poly-silicon nanowires (poly-SiNW) having film thicknesses ranging from 40 – 90 nm were shown [7] to display memristor behavior. Explanation of memristor behavior assumed space charge uniformly distributed over the entire volume of the film, although a physical mechanism by which the space charge is created was not identified. Simulations in fitting of experimental data gave the poly-SiNW band gap of 1.35 eV.

Charge trapping and de-trapping were claimed to be the source of the hysteretic I - V curves. Trapped holes at the SiO_2 /poly-Si interface create positive fixed charges, and are de-trapped with increasing voltage. Instead, QED radiation is the likely mechanism by which the space charge is created.

3. *Light Induced Memristors* Based on the premise that memristive behavior should be common in nanoscale electronics because ionic motion can be induced by the high electrical fields across the memristor, organic materials are shown [8] to emit light, although at very low memristor frequencies. Memristors comprising the metal complex ruthenium (II) tris(bipyridine) with hexafluorophosphate counter ions $[\text{Ru}(\text{bpy})_3]^{2+}(\text{PF}_6^-)_2$ were processed into 120 nm films. LUMO of $[\text{Ru}(\text{bpy})_3]^{2+}(\text{PF}_6^-)_2$ has a bandgap of 3.2 eV and may be excited by QED photons induced in the 120 nm film. Again, the space charge is produced by QED radiation.

The transient current in (Fig. 2A of [8]) is similar to the waveform shown here in Fig. 4. Not shown are solutions having both zero resistance and voltage at some time giving almost a sinusoidal current. Careful inspection showed small perturbations in the current because the voltage and current are not exactly in phase with each other. With larger phase difference, the current will significantly deviate from sinusoidal as shown in Fig. 4. Otherwise, the I - V curve (Fig. 2 of [8]) is similar to Fig. 5. The greatest difference is the very low alternating voltage frequency of 3.3 mHz compared to the 5 kHz frequency assumed in this paper, but the memristor behavior is still displayed.

C. HP Breakthrough

The HP claim [9] of a breakthrough by the discovery of channel formed by the phase change from amorphous to anatase TiO_2 requires more STXM study to establish the channel was not simply an anomaly of electroforming. STXM stands for scanning transmission x-ray microscope. Without switching, more than a single electroformed sample needs to be studied by STXM to determine if the proposed channel as a source of oxygen vacancies is by the fabrication process.

In support of the phase change induced channel, HP presented a thermal analysis using the finite element program CONSOL in the belief the switching caused a hot spot in the memristor. But COMSOL is based on the classical physics assumption that the TiO_2 has specific heat, and therefore is applicable only to the macroscale. QM dismisses classical physics at the nanoscale because the thin film has no specific heat through its thickness.

Indeed, there is no reason Joule heat [9] needs to increase the TiO_2 temperature during switching to produce the oxygen vacancies necessary to explain memristor behavior. Indeed, memristor behavior is observed [5-8] without oxygen vacancies. Instead, the HP memristor is most likely creating QED photons from the Joule heat in the TiO_2 film and producing charge from excitons throughout the memristor including the anomalous phase change region.

D. Phase-Change Materials

QM suggests memristive behavior in PCRAM [10] may not be caused by melting from laser or electrical current pulses. Provided the film is isolated in the sandwich by materials of lower refractive index, melting is precluded by QM. What actually occurs is likely the decrease in amorphous resistance from holes and electrons in excitons created in chalcogenide by the QED induced radiation.

Similarly, the Ovshinsky Effect [11] where thin amorphous chalcogenide films are transformed into semiconductors upon the application of low voltage may actually be caused by excitons created by QED radiation. Ovshinsky's explanation that the decrease in amorphous resistance is caused by redistribution of oppositely charged carriers and not by melting at high temperature is consistent with QM and creating the holes and electrons of excitons by QED radiation.

E. Nanocars

Nanocars including molecular motors are nanostructures that convert absorbed EM energy into mechanical motion. The EM energy may take various forms including light, thermal, Joule heat, and electron beams, e.g., nanocars move by simply heating the substrate. But the mechanism by which EM energy is converted into motion is not well understood.

Molecular dynamics (MD) simulations of heat transfer in nanostructures is used to understand how temperatures cause the observed motions. However, MD simulations based on classical physics by statistical mechanics erroneously allow the atom to have heat capacity. Because of this, the MD simulations [12] show the nanocars distort without moving as the gold surface temperature is increased.

But this is not surprising. Indeed, the nanocars should not move by heating the surface, as we know from common experience when we park our car in a lot with a flat surface on a hot summer day. If heating cars would produce charge, our car could move by interacting with other charged cars. But classical physics precludes charge generation by heating the car. MD of nanocars is no different. Fig. 2 shows classical physics allows the atoms in macroscopic cars to have the same kT energy as the atoms in the molecular prototype. What this means is finite element programs such as COMSOL instead of MD could have been used to show nanocars do not move.

However, QM differs in that the atom lacking heat capacity at the nanoscale conserves the heat from the hot surface by the creation of QED photons that charge the cars by the photoelectric effect, the consequence of which is the nanocars move by electrostatic interactions.

E. Ballistic Heat Transfer

Diffusive heat transport occurs when the device size is far larger than the mean free path of phonons which for most materials is less than about 100 nm. In the diffusive limit, the voltage V is given in terms of the length L of the contact by $V^2 = 4L(T_m^2 - T^2)$ where T_m is the maximum temperature inside the contact and T ambient temperature. For contact lengths < 100 nm, a similar heat transport relation is not yet available because of the difficulty in measuring temperature..

However, experiments [13] claim to measure the temperature inside nanoscale contacts using the magnetic ordering temperature as an embedded thermometer. A single ferromagnetic CoS_2 crystal was used as the sample to which the nanocontacts were made. By assuming the temperature inside the nanocontacts was that of the CoS_2 crystal. magnetometer and resistance measurements clearly showed the temperature T_m of the crystal to increase above ambient and follow the relation $V^2 = C(T_m - T)$ where C is a material constant.

In the experiment, the temperature inside the contact was not measured, but rather that of the temperature in the CoS_2 crystal having a macroscopic size on the order of millimeters. Hence, the contacts were a part of a macroscopic solid, and therefore the temperature measured were those of the CoS_2 and not the nanocontact itself.. Experiments to measure temperatures inside submicron crystals would once and for all establish whether ballistic heat transport or QED heat transfer is operating at the nanoscale.

V. SUMMARY AND CONCLUSIONS

The original paper by Chua and others to date are classical approaches in explaining memristor behavior. Modern day electronics was developed for the macroscale response of resistors, but a QM approach is suggested at the nanoscale where memristive effects are observed.

QED radiation developed for heat transfer in nanostructures based on QM is directly applicable to memristors by precluding any temperature increases to conserve Joule heat. Conservation proceeds by the production of QED photons inside the

memristor that create the space charge of electrons and positive charged holes that produce the memristive effect.

In the HP memristor, the simulations show Joule heat is mostly loss to the surroundings, and only a small fraction η creates excitons. At 5 kHz and mobility 10^{-6} and 10^{-7} $\text{cm}^2/\text{V}\cdot\text{s}$, the fraction η is about 10^{-8} . The photoelectric yield of TiO_2 is of order 10^{-2} , and therefore the exciton yield from QED photons should be much higher. What this means is most QED photons are simply lost from the ultra-thin TiO_2 film.

The mobility of 0.05 $\text{cm}^2/\text{V}\cdot\text{s}$ for electrons and holes in TiO_2 given in the literature appears at least 4 orders of magnitude higher than necessary to fit the HP data. More study is required.

Generally, explanations of memristive effects need not rely on oxygen vacancies, electromigration thinning, unexplained space charge, and the like. Reduced resistance in PCRAM films by high temperature melting from laser and current pulses may be negated by QM. Ovshinsky's redistribution of charge carriers is more likely. Further study is required.

Ballistic heat transport across nanocontacts is negated by QED heat transfer as QM precludes temperature changes.

Memristors have nothing to do with the notion of the missing fourth element necessary to provide completeness for the symmetry of the resistor, capacitor, and inductor. Instead, memristor behavior is simply a QM size effect.

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