Memristors by Quantum Mechanics

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Abstract. Memristor behavior is explained with a physical model based on quantum mechanics that claims charge is naturally created anytime energy is absorbed at the nanoscale. Quantum mechanics requires specific heat to vanish at the nanoscale, and therefore the electrical resistive heating in the memristor cannot be conserved by an increase in temperature. Conservation proceeds by frequency up-conversion of the absorbed energy to produce photons that in submicron thin films have energy beyond the ultraviolet. By the photoelectric effect, the photons create excitons *inside* the memristor that decrease resistance only to be recovered later in the same cycle as the electrons and holes of the excitons are attracted to and destroyed by the polarity of the voltage terminals. Observed memristor behavior is therefore the consequence of excitons being created and destroyed every cycle.

Keywords: memristor, quantum mechanics, quantum electrodynamics.

1 Introduction

In 1971, Chua published a paper [1] claiming a passive two-terminal circuit element existed having a resistance that depended on the time–integral of the current. Based on symmetry arguments alone the notion was held that in electronic circuitry based on the three common elements - the resistor, capacitor, and inductor was incomplete. For completeness, a fourth element [2] called a *memristor* was proposed. But lacking an actual prototype, the memristor lay dormant for almost 40 years until a group [3] at Hewlett-Packard (HP) in 2008 announced the development of a switching memristor based on a thin film of titanium dioxide (TiO_2) sandwiched between platinum (Pt) electrodes.

The HP memristor has a linear relation of resistance to charge provided the current stays within limits [4]. In the OFF state, the memristor has the usual resistance. But oxygen vacancies in the TiO_2 are assumed, the vacancies acting as positive charges that reduce resistance in the ON state. Between OFF and ON states, current flow under positive bias causes the electrons to move to the positive terminal and positive charged vacancies to move toward the negative terminal; while for negative bias, the electrons and positive charges reverse directions. If the bias voltage is set to zero, the current vanishes and the memristor resistance retains the last resistance that it had at the instant the current was stopped. When the current flows again, the resistance of the memristor will be what it was when the current stopped.

The HP memristor [3] is basically a variable resistor dependent on the amount of charge Q transferred. The voltage V across the memristor terminals is,

$$V = I\,M(Q) \quad \text{and} \quad I = dQ/dt \rightarrow Q = \int I dt \qquad (1)$$

where, I is the current and M(Q) the resistance. The charge Q is therefore the time integral of the current I. If M(Q) does not change with charge Q, the resistance R given by Ohms law, i.e., M(Q) = R = V/I. Similarly, the power P dissipated by the memristor is,

$$P = IV = I^2 M(Q) \qquad (2)$$

Currently, HP memristor theory assumes positive charge from oxygen vacancies is the source of switching, but the theory is phenomenological lacking a physical basis to allow extensions to other memristors without vacancies. In fact, many experiments reported over the past 50 years show memristor behavior, e.g., sandwiched molecular layers [5] between gold electrodes, and modification [2] of electrical conduction in solid electrolytes, all of which exclude positive charge in vacancies. But sandwiched material between electrodes is not even necessary. Indeed, memristor behavior is observed in a single material without electrodes, e.g., gold [6] and silicon [7] nanowires. Lacking vacancies, explanations of memristor behavior assume the presence of space charge, but the mechanism by which the space charge is produced is not identified. Space charge is also claimed [8] to explain light emission from organic memristors. However, vacancies or space charge need not be assumed.

Memristor behavior relying on the creation of charge Q is a natural consequence of QM anytime EM energy is absorbed at the nanoscale, e.g. in heat transfer [9] by QED induced radiation from the EM confinement of photons. QM stands for quantum mechanics, EM for electromagnetic, and QED for quantum electrodynamics.

Memristors are submicron, and therefore QM is required in any explanation of charge creation, say in carbon nanotubes (CNTs). Although CNT diameters are submicron, QM was thought [5] invalidated by CNTs having supramicron lengths. However, the EM confinement of QED photons in any one dimension [9] is sufficient to justify the validity of QM, say across the CNT diameter. Moreover, the applicability of QM to memristors is supported by the fact that only at the nanoscale [3] is the memristor behavior detectable. In contrast, supramicron diameter memristors behave just like ordinary resistors where resistance is equal to the voltage divided by the current. Electronic circuitry was originally developed at the macroscale with resistor diameters too large to notice the QM of memristor behavior.

2 Purpose

Propose the charge in memristors is a QM effect that produces QED radiation from the conservation of resistive heating that otherwise is conserved by an increase in temperature. QED radiation creates excitons *inside* the memristor. Charged vacancies are not necessary because excitons are created and destroyed in the switching cycle.

3 Theory

Memristors are generally thin films and nanowires. Thin films having material of thickness d sandwiched between metal electrodes while nanowires of a single material having diameter D and length L. QED radiation creates excitons comprising mobile holes and electrons *inside* the memristor that decrease the resistance, but the resistance is promptly recovered as the holes and electrons are destroyed upon being neutralized at the voltage terminals.

3.1 QM Restrictions

To understand how QED radiation is produced in memristors, consider the QM restriction on heat capacity in conserving resistive heating by an increase in temperature. Unlike classical physics, the specific heat capacity of the atom by QM depends on its EM confinement. At 300 K, the Einstein-Hopf relation giving the average Planck energy for the harmonic oscillator in relation to kT and thermal wavelength λ_T is shown in Fig. 1.

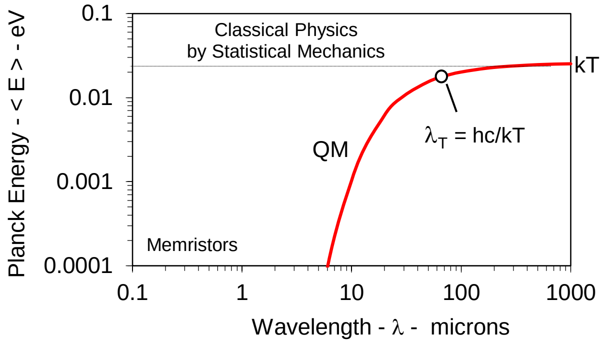

Fig. 1 Classical and QM Oscillators - Heat Capacity at 300 K

Unlike classical oscillators having kT energy at all wavelengths, QM oscillators only allow kT energy for $\lambda > \lambda_T$ and restrict kT for $\lambda < \lambda_T$. At ambient temperature, $\lambda_T \approx 50$ microns. Fig. 1 shows the specific heat capacity is less than kT for $\lambda < \lambda_T$ with kT energy available only for $\lambda > \lambda_T$. For memristors having $\lambda < 1$ micron, QM by requiring specific heat to vanish precludes any increase in memristor temperature upon the absorption of EM energy.

3.2 QED Confinement

Memristors lack specific heat and cannot conserve absorbed EM energy by an increase in temperature. Instead, conservation may only proceed by the QED induced frequency up-conversion of the absorbed EM energy to the TIR confinement frequency of the memristor. TIR stands for total internal reflection. Since memristors have high surface to volume ratios, the absorbed EM energy is confined by TIR almost entirely in the memristor surface. The TIR confinement is momentary and occurs only upon absorption of EM energy, and therefore, the TIR confinement effectively sustains itself.

Similar to creating QED photons of wavelength λ by supplying EM energy to a QM box with sides separated by $\lambda/2$, the absorbed EM energy is frequency up-converted to the characteristic dimension D_C of the memristor. The QED photon energy E and frequency f are:

$$E = hf \quad f = c/\lambda \quad \lambda = 2nD_C \tag{3}$$

where, h is Planck's constant, c the velocity of light, and n the refractive index of the memristor. For memristors of thin films and nanowires, the characteristic dimensions D_C are the thickness d and diameter D, respectively.

3.3 QED photons and Rate

Classical heat transfer conserves absorbed EM energy by an increase in temperature, but is not applicable to memristors because of QM restrictions on specific heat. Instead, the EM dissipative power P is conserved by creating number N_P of QED photons *inside* the memristor having Planck energy E at the photon rate dN_P/dt,

$$\frac{dN_P}{dt} = \frac{P}{E} = \frac{I^2 M(Q)}{E} \tag{4}$$

Only a fraction η of QED radiation creates excitons, the remainder (1-η) lost to the surroundings. By the photoelectric effect, the rate dNex/dt of excitons, each exciton comprising an electron and hole is,

$$\frac{dNex}{dt} = \eta e \frac{dN_P}{dt} \tag{5}$$

where, e is the electron charge. The charge Q produced,

$$Q = \int \frac{dNex}{dt} dt = \eta e \int \frac{dN_P}{dt} dt \tag{6}$$

3.4 QED Photons, Excitons, and Charge

The creation of charge Q *inside* the memristor depends on the electrical dissipative power $P = I^2 M(Q)$. If current $I > 0$, QED photons are produced at rate $dN_P/dt > 0$, and therefore charge Q is produced. Note that reversal of current does not alter the rate dN_P/dt of QED photons created and charge Q produced. But if current $I = 0$, QED photons are not created and no charge is produced.

Charge Q is produced from excitons *inside* the memristor by QED photons having Planck energy greater than the band gap. The excitons comprising electron-hole pairs form as electrons are excited from the filled valence band to the conduction band leaving a mobile positive charged hole corresponding to the charge Q.

The memristor resistance M(Q) depends on the electrical conductivity σ of the material given by the number density of electrons N_E and holes N_H,

$$\sigma = e(N_E\mu_E + N_H\mu_H) \quad (7)$$

where, μ_E and μ_H are the electron and hole mobility. Electron-hole pairs may recombine, to form another photon having lower energy than the QED photon. But this is unlikely in memristors because the electrons separate from the holes under the high field $F = \Delta V/d$ across the memristor, where ΔV is the voltage drop and d is thickness. Hence, the N_H increases. If the current stops, the hole charge Q_H is trapped. Upon the application of the bias voltage, the resistance M(Q) begins with the value it had when the current stopped, i.e., the memristor remembers the last resistance.

3.5 Charging by QM

The charging Q of the memristor occurs during the sinusoidal voltage if $|I| > 0$. Excitons form in proportion to the fraction ηP of QED photons absorbed, the positive charged holes Q_H and negative electrons Q_E under the high electric field then promptly drift toward the opposite polarity voltage terminals as shown in Fig. 2.

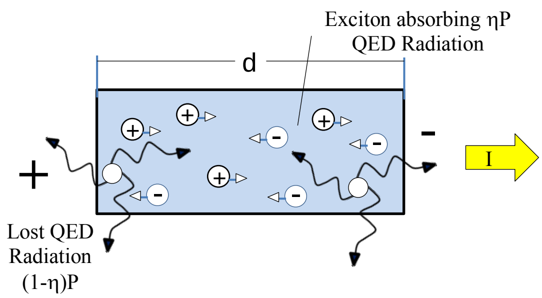

Fig. 2 Memristor - Charge Distribution – Positive Voltage Bias

Conservation of electron charge Q_E within the memristor gives,

$$\frac{dQ_E}{dt} = \frac{\eta P}{E} + \frac{I}{e} - Q_E\frac{\mu_E F}{d} \quad (8)$$

where, F is the electrical field across the memristor. Charge Q_E in electrons is assumed uniformly distributed over the thickness d having units of Q_E / d electrons/m. Similarly, conservation of positive hole charge Q_H is,

$$\frac{dQ_H}{dt} = \frac{\eta P}{E} - Q_H\frac{\mu_H F}{d} \quad (9)$$

Eqns. 8 and 9 are a set of simultaneous non-linear DE's, but even numerical solutions are difficult.

Taking $F = V/d$ and noting terms $\eta P/E$ and $Q_E \mu_E F/d$ are small compared to I/e allow the simplification,

$$Q_E \approx \int I dt \quad \text{and} \quad \frac{dQ_H}{dt} = \frac{\eta P}{E} - \frac{\mu_H V}{d^2} Q_H \tag{10}$$

The hole Q_H solution is,

$$Q_H = Q_{HO} + \frac{\eta P}{E} \frac{d^2}{\mu V} \left[1 - \exp\left(-\frac{\mu_H V}{d^2} t \right) \right] \tag{11}$$

where, Q_{HO} is the hole charge at current I = 0. The voltage V is sinusoidal, $V = V_O \sin\omega t$, where ω is the circular frequency.

The resistance R is,

$$R = \rho \frac{d}{A} = \frac{d}{A} \frac{1}{\sigma} = \frac{d}{A} \frac{1}{e(\mu_E Q_{EO} + \mu_H Q_{HO})/Ad} \approx \frac{d^2}{2e\mu_H Q_H} \tag{12}$$

where, ρ is the resistivity. For simplicity, the conductance σ assumes the electron density Q_E contribution may be represented by that of the Q_H holes. Note the conductance σ requires units of per unit volume, where volume is Ad and A is memristor area. The initial resistance R_O corresponds to hole charge Q_{HO},

$$Q_{HO} = \frac{d^2}{2e\mu_H R_O} \tag{13}$$

The current I is,

$$I = \frac{V}{R} = \frac{V_O \sin\omega t}{R} \tag{14}$$

4 Application

4.1 HP Memristor

The HP memristor [3] is comprised of TiO_2 having thickness d = 50 nm sandwiched between t = 5 nm electrodes, one Pt and the other Ti. The area A is not known and is assumed to be 200 x200 nm^2.

QED Photons and Rate The power $P = I^2R$ may be determined from $I_o = V_o/R_o = 10$ mA and $V_o = 1V$. Hence, $R_o = 100\ \Omega$ and P = 10 mW. From Eqn. 3, the QED photons have Planck energy E = hc/2nd. The refractive index n of TiO_2 depends on structure: for rutile and anatase, n = 2.7 and 2.55, respectively. For TiO_2 thickness d = 50 nm, the Planck energies E of which are 4.6 and 4.87 eV, respectively. Eqn. 4 gives the respective QED photon rates dN_P/dt as 1.36×10^{16} and $1.28 \times 10^{16}\ s^{-1}$.

Exciton and Holes The bandgap of TiO_2 is about 3.2 eV, and therefore QED photons having Planck energies > 4.6 eV produces excitons, i.e., every QED photon produces one exciton. Numerical solutions show the fraction η of QED radiation producing excitons is near unity. For 1 GHz cycling, the time dependence of resistance R and current I for 2 cycles is illustrated in Fig. 3. Maximum current I = 10 mA and voltage V = 1 V. The I-V curves for the first 2 switching cycles are shown in Fig. 4.

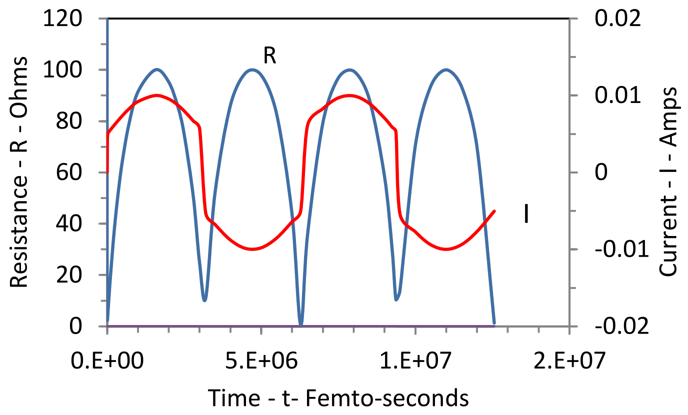

Fig. 3 Transient Resistance R and Current I at 1 GHz

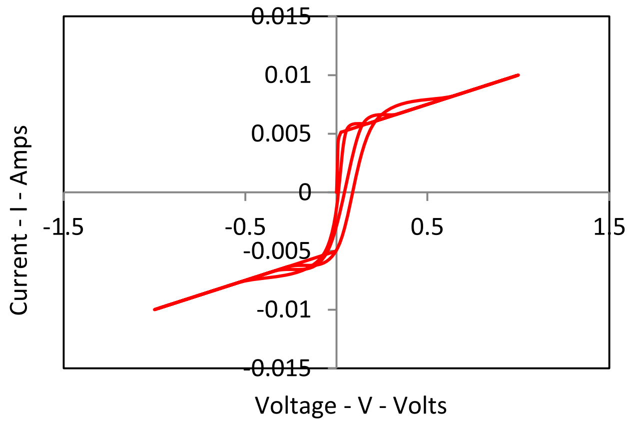

Fig. 4 Memristor I-V Curve – 2 Cycles at 1 GHz
$\mu_E = \mu_H = 500\ cm^2/V\text{-}s$, P = 10 mW, V= 1V, $R_O = 100\ \Omega$, I = 10 mA
E = 4 eV, A = 200x200 nm^2, d = 50 nm

In Fig. 4 the I-V curves assume the fraction η of QED radiation is near unity. Solutions for η << 1 show the I-V curves approach a straight line, i.e., a linear resistance if QED radiation is not available to create holes in the TiO_2 body of the memristor. .

5 Summary and Conclusions

The original paper by Chua and numerous other papers to date are classical approaches in explaining memristor behavior. Modern day electronics was developed based on macroscale response of resistors, but a QM approach is suggested at the nanoscale where memristive effects are observed.

QED radiation developed for heat transfer in nanostructures based on QM is directly applicable to memristors by precluding any temperature increases to conserve electrical resistive heat. Conservation proceeds by the production of QED photons *inside* the memristor that create excitons, the electrons and positive charged holes of which produce the memristive effect.

In the HP memristor, the electrical resistive heat is almost entirely converted to excitons, and only a small fraction is lost to the surroundings.

Generally, explanations of memristive effects need not rely on oxygen vacancies, electromigration thinning, unexplained space charge, and the like.

Provided QM is included in the argument, HP and other memristors as the missing fourth element provide completeness for the symmetry of the resistor, capacitor, and inductor.

More study is required to extend the application of QED radiation to both HP and other memristors. Numerical simulations are planned based on the QM response without simplifying approximations.

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